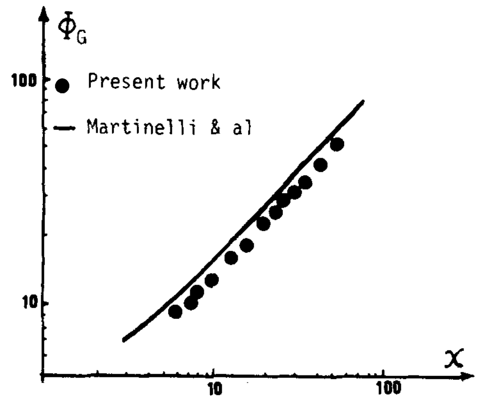

Figure 8. Comparison of experimental results in plug flow with Martinelli's data.

measurements of wall friction in a two-phase flow. By more elaborate statistical processing of the signal received from the probes, we hope to obtain more precise data about the structure and transfer properties of such flows near the wall as has already been done in a one-phase turbulent flow.

NOTATION

C_o	= concentration of active ions, mol/cm^3
D	= internal diameter of the circular duct, mm
$\mathcal{D}$	= diffusion coefficient of active ion, m^2/s
F	= Faraday's constant
$I, I_1 - I_2$	= currents delivered by single probe, double probe, A
K	= mass transfer coefficient $K = I(nFC_oLl)^{-1}$
L	= duct length from gas injector to the measuring probe, mm
l, L	= width and length of rectangular probe, mm
n	= number of electrons involved in the electrochemical reaction
p	= pressure, Pa
S	= wall shear, s^{-1}
T	= temperature
U	= velocity, m/s

Greek Letters

ρ	= Density kg/m^3
μ,ν	= Dynamic viscosity Pl; Kinematics viscosity m^2/s
τ	= Wall friction, $\overline{}$ time average, | | absolute value
δ_c, δ^+	= Concentration boundary layer thickness, dimensionless value of δ_c
χ, ϕ_G	= Martinelli's parameters

$$\chi^2 = \left(-\frac{\partial p}{\partial z}\right)_{L,o} \bigg/ \left(-\frac{\partial p}{\partial z}\right)_{G,o}$$

$$\phi_G^2 = \left(\frac{4\tau}{D}\right) \bigg/ \left(-\frac{\partial p}{\partial z}\right)_{G,o}$$

Subscripts

G,GS	= gas, gas standard conditions
L	= liquid
o	= one-phase flow

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Dimensionless Presentation of Performance Data for Fans and Blowers

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The purpose of this paper is to show how dimensional analysis and simple physical arguments can be used to extend the utility of available performance data for such low-pressure fans and blowers as those used in ventilation systems. Our development arose in response to our desire to reduce the numbers of tables and graphs now supplied for describing the performance of a blower series, and to our need to extend the data supplied by manufacturers to operating conditions beyond those described.

The primary result of our development is to show that, for any one device, all properly-scaled measures of performance depend essentially on only one parameter, the scaled throughput, Q^*

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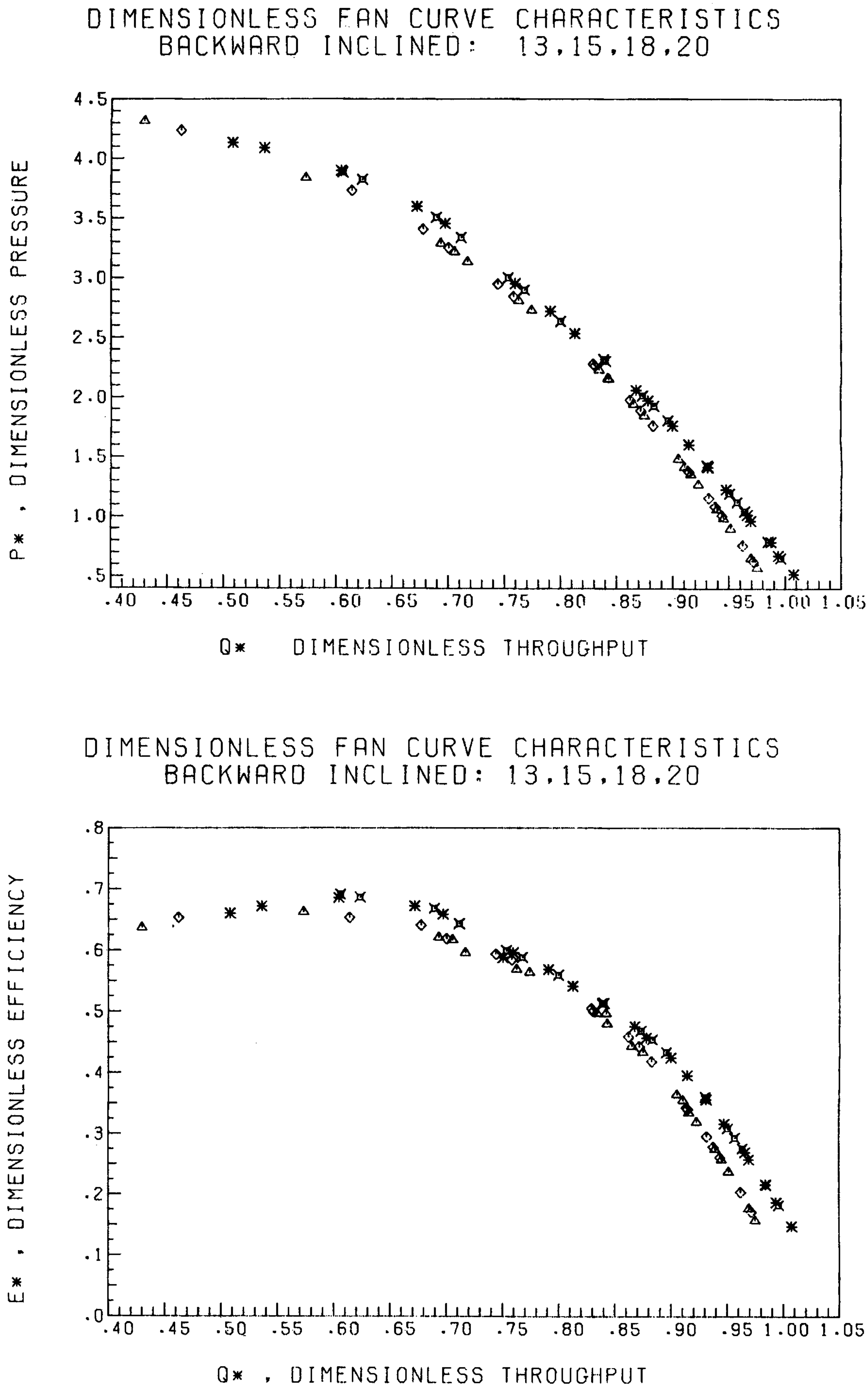


Figure 1. Dimensionless fan curves for Trane Co., belt drive, backward inclined fans. Static pressures vary from 93.4 to 498 N/m^2; rotor speeds, 57.2 to 250 rad/s; power, 59.7 to 3,330 W; and the delivery, 0.396 to 3.175 m^3/s. △ = the 0.343-m wheel diameter; ◇ = 0.381 m; ⌘ = 0.464 m; and ✳ = 0.508-m wheel diameter fan. The curves for the 0.343 and 0.381 m fans are indistinguishable as are the curves for the 0.464 and 0.508 m.

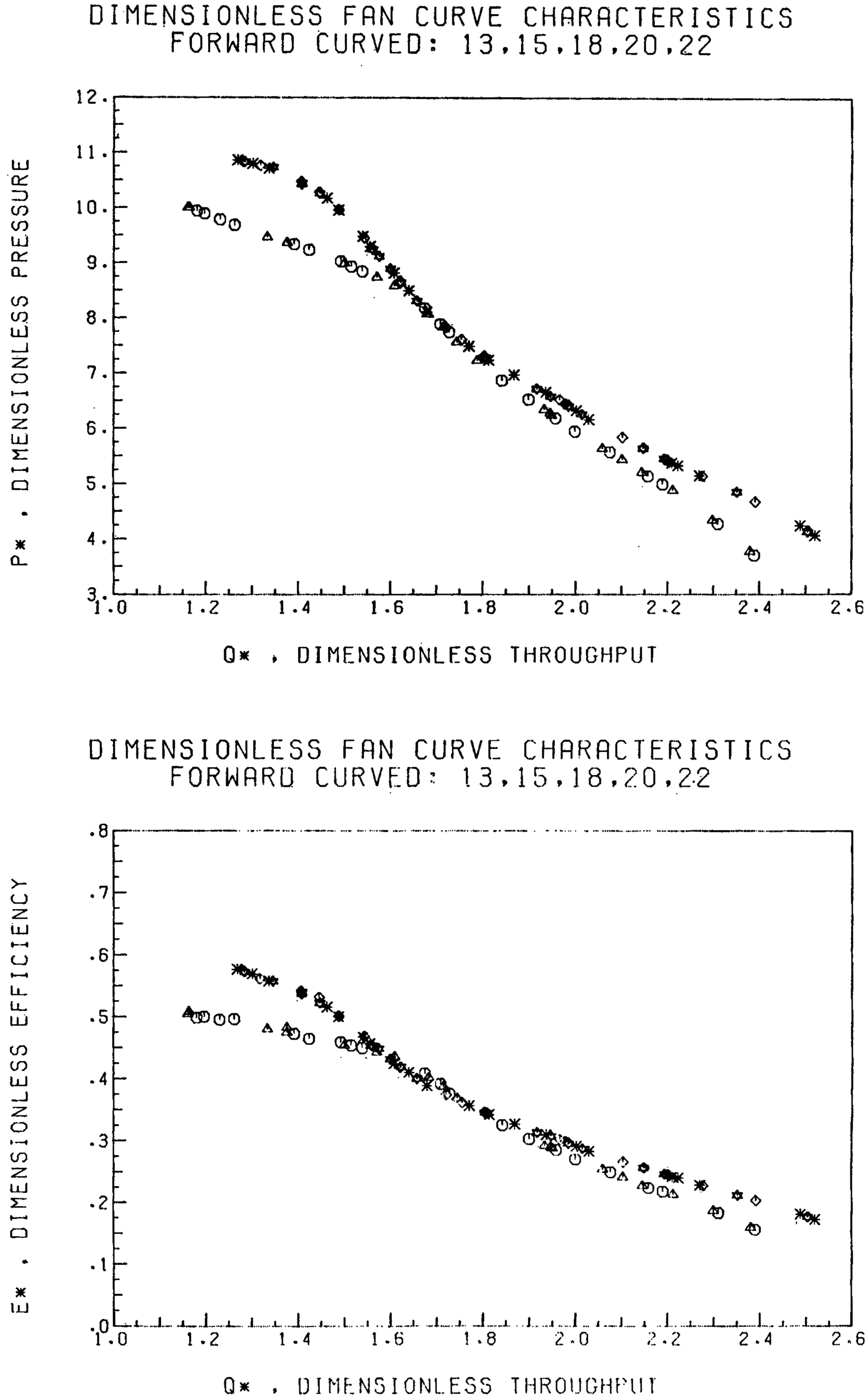
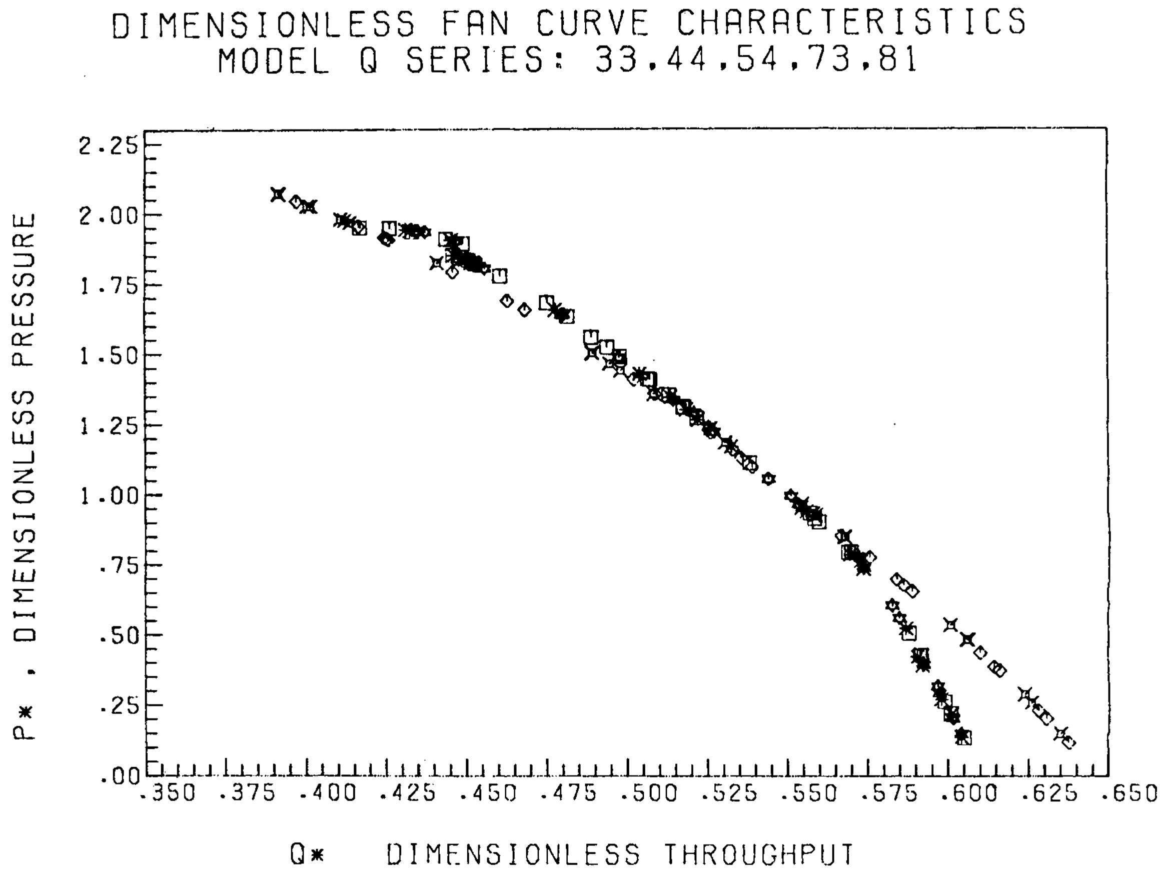


Figure 2. Dimensionless fan curves for Trane Co., belt drive, forward curved fans. The range of values of operating conditions are: static pressure, 93.4 to 498 N/m^2; rotor speed, 56.8 to 250 rad/s; power, 112 to 3,670 W; and air delivery, 0.396 to 3.473 m^3/s. octagons = 0.343-m wheel diameter fan; triangle = 0.381-m fan; diamond = 0.464 m fan; asterisk = 0.508-m fan; and star = 0. 565-m diameter fan.



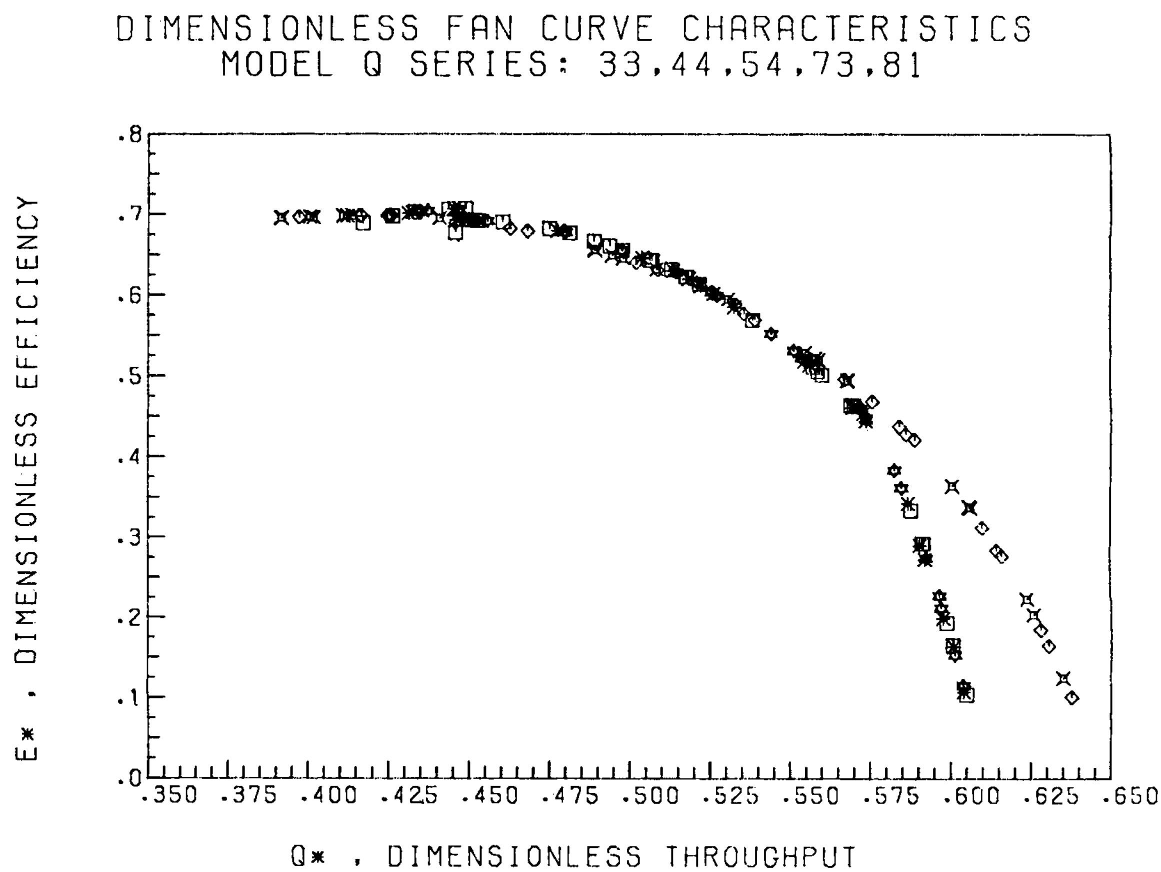


Figure 3. Dimensionless fan curves for Trane Co., Model *Q*, axial-flow fans. Values of the operating conditions vary over a large range: static pressure, 62.3 to 2,864 N/m^2; rotor speed, 22.8 to 256 rad/s; power, 544 to 257,590 W; and air delivery, 3.68 to 74.1 m^3/s. diamond = 0.838-m wheel diameter fan; crossed square = 1.130-m fan; asterisk = 1.378-m fan; star = 1.854-m fan; and square = 2.051-m diameter fan. Performance curves for five fan sizes form a single curve over a very wide range of operating conditions.

$$Q^* \equiv Q/ND^3 \quad (1)$$

As examples:

$$\frac{\Delta \mathcal{P}}{\rho N^2 D^2} = f(Q^*) \quad (2)$$

and

$$\frac{Q\Delta \mathcal{P}}{HP} = g(Q^*) \quad (3)$$

Clearly the functions f and g are related by the macroscopic energy balance, and we shall therefore concentrate our attention on Eq. 2. We begin with experimental verification of this relation and a demonstration of its utility. We then follow with a brief justification.

In principle, one can use Eqs. 2 and 3 as the basis for describing a series of geometrically identical devices of differing size with just two relations; but in practice, few such production series exist. However, some are quite close. As is normally done it will be possible to neglect changes in air density, since these are normally small compared to uncertainties in available data, and use the properties of standard air (294 K [70°F], 1.013×10^5 N/m^2 [1 atm.]). Specifically, we suggest that Eqs. 1 and 2 should hold quite accurately for any single model of fan. For this special case, these correlations take the simplified form:

$$\Delta \mathcal{P}/N^2 = fn(Q/N) \quad (4)$$

$$E^* = fn(Q/N) \quad (5)$$

For fans of a given manufacturer's series, one could not expect complete dimensionless similarity, but one in fact hopes that differences would be small.

Tests of Eqs. 2 and 3 in Figures 1 to 3 are quite satisfactory; all manufacturers data for each unit fall on a simple curve and in several cases we have found that data for up to four sizes of fans form one curve. Such curves for blowers of the backward-inclined series, the forward-curved series, and the axial-flow series express all necessary information in two simple figures: head and efficiency in terms of volumetric flow rate and fan speed or power supplied to the rotor. The points plotted were arbitrarily selected from performance tables supplied by Trane Co. We have obtained equally impressive results with other fans and have found no significant discrepancies. These equations are, therefore, successful and it remains to examine their basis in transport theory.

We begin by recognizing that performance of this system is described in the absence of compressibility effects by the equations of continuity and motion for incompressible fluids and pertinent boundary conditions. Of these latter the most important for our purposes in the specification of flow rate through the fan. We thus obtain (Bird et al., 1960):

$$\text{(continuity)} \qquad \nabla^* \cdot \mathbf{v}^* = 0 \quad (6)$$

$$\text{(motion)} \qquad \frac{D\mathbf{v}^*}{Dt^*} = \frac{1}{Re}[\nabla^* \cdot \nabla^* \mathbf{v}^*] - \nabla^* \mathcal{P}^* \quad (7)$$

At the inlet

$$Q^* = Q/ND^3 \quad (8)$$

Other boundary conditions serve primarily to establish fan geometry and need not be considered explicitly.

It follows from this description that both the pressure distribution $\mathcal{P}^*(\mathbf{r}/D)$ and the velocity distribution $\mathbf{v}^*(\mathbf{r}/D)$ are functions only of Re and Q^*. It follows then that

$$\frac{\Delta \mathcal{P}}{\rho N^2 D^2} = F(Re, Q^*) \quad (9)$$

which is as far as dimensional analysis can take us. Similarly, dimensionless efficiency is a function of these same variables:

$$\frac{Q^* \Delta \mathcal{P}^*}{N_p} = G(Re, Q^*) \quad (10)$$

To proceed further, one must recognize that the Reynolds numbers in fans and blowers are quite high and that interaction between rotor and air is primarily via form drag rather than friction drag. Under these circumstances $\mathbf{v}^*(\mathbf{r}/D)$ becomes Reynolds-number-insensitive, except in thin boundary layers on solid surfaces which do not appreciably affect form drag. It follows that Eq. 9 reduces to Eq. 2, and Eq. 10 reduces to Eq. 3. This situation is analogous to the constancy of the friction factor in real pipes and the power number in agitated tanks at sufficiently high Reynolds number.

NOTATION

D = rotor diameter (L)
E^* = dimensionless efficiency, $Q\Delta\mathcal{P}/HP$
f = experimentally determined function
F = experimentally determined function
g = experimentally determined function
$\mathbf{g}$ = gravitational acceleration (Lt^{-2})
G = experimentally determined function
HP = power supplied to the rotor (FLt^{-1})
N = rate of rotation of the rotor (t^{-1})
N_p = dimensionless power number, $HP/\rho N^3 D^5$
p = fluid pressure (FL^{-2})
$\Delta\mathcal{P}$ = increase in static pressure produced by the device, $p - \rho(\mathbf{g} \cdot \mathbf{r})$
$\mathcal{P}^*$ = dimensionless static pressure, $\Delta\mathcal{P}/\rho N^2 D^2$
Q = volumetric throughput rate ($L^3 t^{-1}$)
Q^* = dimensionless throughput rate, Q/ND^3
$\mathbf{r}$ = vectorial distance from coordinate origin (L)
Re = Reynolds number, $D^2 N\rho/\mu$
t = time since start of operation
t^* = dimensionless time, tV/D
$\mathbf{v}$ = fluid velocity (Lt^{-1})
$\mathbf{v}^*$ = dimensionless fluid velocity, $\mathbf{v}/ND$
V = characteristic velocity for the system (Lt^{-1})
∇ = $\frac{\partial}{\partial \mathbf{r}}$, gradient operator ($L^{-1}$)
∇^* = dimensionless gradient operator, $D\nabla$
μ = viscosity ($ML^{-1}t^{-1}$)
ρ = density of the gas (ML^{-3})

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